STRUCTURE CONSTANTS IN THE $N = 1$ SUPER-LIOUVILLE FIELD THEORY

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Abstract

The symmetry algebra of $N = 1$ Super-Liouville field theory in two dimensions is the infinite dimensional $N = 1$ superconformal algebra, which allows one to prove, that correlation functions, containing degenerated fields obey some partial linear differential equations. In the special case of four point function, including a primary field degenerated at the first level, this differential equations can be solved via hypergeometric functions. Taking into account mutual locality properties of fields and investigating s- and t- channel singularities we obtain some functional relations for three- point correlation functions. Solving this functional equations we obtain three-point functions in both Neveu-Schwarz and Ramond sectors.

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1 Introduction

Attempts to achieve deeper understanding of two dimensional quantum (super-) gravity [1]-[7] are mainly motivated by the fact, that these theories appear to be one of the most important building blocks of noncritical (super-) strings [8, 9].

The (super-) Liouville field theory, which is the effective theory of (super-) gravity in two dimensions, has an infinite dimensional (super-) conformal invariance inherited from the initial general (super-) covariance. Therefore, it might be possible to use powerful methods of two dimensional Conformal Field Theory (CFT) to investigate these theories. The main difficulty in this direction is due to the facts, that the physical Hilbert space of (super-) Liouville field theory contains infinite continuum set of primary states and the local field↔state correspondence, which is usual in CFT, is problematic. As has been shown in [12] the correlation functions in Liouville field theory can be calculated using a technique, quite similar to the Coulomb gas representation in ordinary "minimal" models of CFT [13], provided some on mass-shell type conditions are satisfied. In [14, 15], [16] the three point functions of exponential fields in Liouville field theory have been calculated first in above mentioned on mass-shell case, after which a general expression has been conjectured. But there is another method of calculating three point functions in CFT too, which has been used successfully for several models of CFT in [17, 18]. The method is the following. As it is well known [10], correlation functions, containing at least one degenerated field, obey some linear differential equations. In many interesting cases, when degeneration takes place at low levels, four-point correlation functions can be obtained directly solving these differential equations, thus avoiding Coulomb gas type representations. The latter is essential, because neutrality condition, necessary for having Coulomb gas representation, touches all the fields entering correlation function contrary to the degeneracy condition, which is related to a separate field only. This is the reason why investigating four point functions with the help of Coulomb gas representation one obtains relations only for on mass-shell three point functions, while investigation of four point functions, containing a single degenerated field, leads to some nontrivial func-

tional relations for unconstrained three point functions. In the cases of minimal models these relations are reduced to recurrent equations, solving which one obtains three point functions [17], [18]. In the case of Liouville field theory, solving above mentioned functional relations, all the results, conjectured in [15, 16] are reobtained in [19]. In this work, the same method is used in the case of $N = 1$ Supper- Liouville Field theory (SLFT). The further part of this paper is organized as follows. In sec.2 we present a brief review on $N = 1$ SLFT. In sec.3 it is shown that four point correlation functions, including a Ramond field degenerated at second level, obey some linear differential equation. Solving these equations and taking into account locality condition we express four point correlation functions via hypergeometric functions. In sec.4 investigating s- and t-channel singularities of four point correlation functions obtained in sec.3, some functional relations for structure constants are derived. Solving these functional relations all three point functions of exponential fields and reflection amplitudes are calculated in both Neveu-Schwarz and Ramond sectors.

The results of this work have been previously reported in [20]

2 $N = 1$ Super-Liouville, as a Two Dimensional Superconformal Field Theory

The Super-Liouville field theory is a supersymmetric generalization of the bosonic Liouville theory, which is known to be the theory of matter induced gravity in two dimensions. Similarly SLFT describes 2d supergravity, induced by supersymmetric matter. To obtain Super-Liouville action, one can simply "supersymmetrize" the bosonic Liouville action. The answer reads

$$S_{SL} = \frac{1}{4\pi} \int \hat{E} \left[\frac{1}{2} D_\alpha \Phi_{SL} D^\alpha \Phi_{SL} - Q \hat{R} \Phi_{SL} - 4\pi i \mu e^{b\Phi_{SL}} \right], \tag{1}$$

where Φ_{SL} is Liouville superfield, D^α, D_α are superderivatives, $\hat{Y}$ and $\hat{E}$ are supercurvature and superdensity of the background supermanifold (see [5]). The condition, that

cosmological term $\mu \exp b\Phi_{SL}$ has correct dimension $(1/2, 1/2)$ leads to the following relation between "background charge" Q and the coupling constant b

$$Q = b + \frac{1}{b}. \tag{2}$$

One of the most important properties of the action (2.1) is its superconformal invariance. To describe superconformal symmetry and its consequences more explicitly, let us consider SLFT on the superplane, with coordinates $(Z, \bar{Z}) = (z, \bar{z}, \theta, \bar{\theta})$, where $z, \bar{z}$ are complex coordinates on plain, and $\theta, \bar{\theta}$ are corresponding Grassmanian coordinates (for topologically nontrivial supermanifolds such choice of "flat" coordinates can be achieved for every coordinate patch separately, using superdiffeomorfism and super- Weyl transformations). Superconformal transformations in SLFT are generated by the super energy-momentum tensor $\hat{T} = S + 2\theta T$ (T is the ordinary energy-momentum tensor and S is a spin 3/2 conserved current)

$$\hat{T} = -\frac{1}{2} D\Phi_{SL}\partial\Phi_{SL} + \frac{Q}{2} D\partial\Phi_{SL}, \tag{3}$$

where $D = \partial/\partial\theta + \theta\partial/\partial z$ is the covariant derivative. As in every superconformal field theory [21] there are two kinds of primary fields in SLFT: Neveu-Schwarz superfields

$$\Phi_\alpha(Z, \bar{Z}) = \phi_\alpha(z, \bar{z}) + \theta\psi_\alpha(z, \bar{z}) + \bar{\theta}\bar{\psi}_\alpha(z, \bar{z}) - \theta\bar{\theta}\tilde{\phi}_\alpha(z, \bar{z}) \sim e^{\alpha\Phi_{SL}} \tag{4}$$

with dimensions

$$\Delta_\alpha = \frac{1}{2}\alpha(Q - \alpha), \tag{5}$$

(this fields are local with respect to fermionic current $S(z)$) and Ramond fields

$$R^\epsilon_\alpha = \sigma^{(\epsilon)}\phi_\alpha \sim \sigma^{(\epsilon)} e^{\alpha\phi_{SL}}, \tag{6}$$

where $\sigma^{(\epsilon)}$ are so called twist fields (they are quite similar to the spin and disorder fields in 2d Ising model) with dimension 1/16, so that the total dimension of R^ϵ_α is

$$\Delta_{[\alpha]} = \frac{1}{16} + \frac{1}{2}\alpha(Q - \alpha). \tag{7}$$

The characteristic feature of Ramond fields is their nontrivial (Z_2) monodromy with respect to fermionic current $S(z)$. All the other fields of the theory can be obtained

from Neveu-Schwarz (Ramond) primary fields via the action of Neveu-Schwarz (Ramond) algebra generators L_n, S_m, $n \in Z, m \in Z+1/2$ ($n \in Z, m \in Z$), which are the Lorants coefficients of the energy-momentum tensor $T(z)$ and fermionic current $S(z)$ respectively. The commutation relations of the Neveu-Schwarz-Ramond algebra have the form

$$\{S_k, S_l\} = 2L_{k+l} + \frac{\hat{c}}{2}(k^2 - 1/4)\delta_{k+l,0}, \tag{8}$$

$$[L_n, S_k] = \frac{1}{2}(n-2k)S_{n+k}, \tag{9}$$

$$[L_n, L_m] = (n-m)L_{n+m} + \frac{\hat{c}}{8}(n^3 - n)\delta_{n+m,0}. \tag{10}$$

In our case of SLFT the central charge $\hat{c}$ of NSR algebra is equal to

$$\hat{c} = 1 + 2Q^2. \tag{11}$$

Zero modes of fermionic currents S_0 and $\bar{S}_0$ act on Ramond fields R^{ϵ}_{α} as follows:

$$S_0 R^{\epsilon}_{\alpha} = i\beta e^{-i\epsilon\pi/4} R^{-\epsilon}_{\alpha}, \tag{12}$$

where

$$\beta = \frac{1}{\sqrt{2}}(\frac{Q}{2} - \alpha). \tag{13}$$

Further discussion is closely parallel to that of [17]. Due to Liouville reflection the fields Φ_α and $\Phi_{Q-\alpha}$ are not independent (the same is true for R^{ϵ}_{α} and $R^{\epsilon}_{Q-\alpha}$) so that we can restrict the variation range of parameter α to be $\alpha \leq Q/2$. It follows from superconformal symmetry, that the two-point functions have the form:

$$\langle \Phi_{\alpha_1}(Z_1, \bar{Z}_1)\Phi_{\alpha_2}(Z_2, \bar{Z}_2)\rangle = \Delta(\alpha_1 - \alpha_2)(Z_{12}\bar{Z}_{12})^{-2\Delta_{\alpha_1}}, \tag{14}$$

$$\langle R^{\epsilon_1}_{\alpha_1}(z_1, \bar{z}_1)R^{\epsilon_2}_{\alpha_2}(z_2, \bar{z}_2)\rangle = \delta^{\epsilon_1,\epsilon_2}\Delta(\alpha_1 - \alpha_2)(z_{12}\bar{z}_{12})^{-2\Delta_{[\alpha_1]}}, \tag{15}$$

where $\Delta(\alpha_1 - \alpha_2)$ is some generalized function, which, according to the superconformal symmetry is nonzero only if $\alpha_1 - \alpha_2 = 0$, and the superdistance $Z_{12} = z_{12} - \theta_1\theta_2$. Only at the end of the paper we'll restore the proportionality coefficients in (4), (6) and extend the variation range of parameter α in order to obtain so called "reflection amplitudes".

As usual, the form of three point functions are restricted up to some numerical coefficients by the superconformal invariance:

$$\langle \Phi_{\alpha_1}(Z_1,\bar{Z}_1)\Phi_{\alpha_2}(Z_2,\bar{Z}_2)\Phi_{\alpha_3}(Z_3,\bar{Z}_3)\rangle =$$
$$= \left\{C_{(\alpha_1),(\alpha_2),(\alpha_3)} + \Theta\bar{\Theta}\tilde{C}_{(\alpha_1),(\alpha_2),(\alpha_3)}\right\}(Z_{12}\bar{Z}_{12})^{\lambda_3}(Z_{13}\bar{Z}_{13})^{\lambda_2}(Z_{23}\bar{Z}_{23})^{\lambda_1}, \quad (16)$$

$$\langle \Phi_{\alpha_3}(Z_3,\bar{Z}_3)R^{\epsilon_1}_{\alpha_1}(z_1,\bar{z}_1)R^{\epsilon_2}_{\alpha_2}(z_2,\bar{z}_2)\rangle = (z_{12}\bar{z}_{12})^{\lambda_3}(z_{13}\bar{z}_{13})^{\lambda_2}(z_{23}\bar{z}_{23})^{\lambda_1}\times$$
$$\times\left[\delta_{\epsilon_1,\epsilon_2}\left(C^{\epsilon_1}_{[\alpha_1],[\alpha_2],(\alpha_3)} + \tilde{C}^{\epsilon_1}_{[\alpha_1],[\alpha_2],(\alpha_3)}\tilde{C}^{\epsilon_1}_{[\alpha_1],[\alpha_2],(\alpha_3)}\theta_3\bar{\theta}_3|\frac{z_{31}z_{32}}{z_{12}}|\right)+\right.$$
$$\left.+\delta_{\epsilon_1+\epsilon_2,0}\left(d^{\epsilon}_{[\alpha_1],[\alpha_2],(\alpha_3)}\left(\frac{z_{31}z_{32}}{z_{12}}\right)^{1/2}\theta_3 + \bar{d}^{\epsilon}_{[\alpha_1],[\alpha_2],(\alpha_3)}\left(\frac{\bar{z}_{31}\bar{z}_{32}}{\bar{z}_{12}}\right)^{1/2}\bar{\theta}_3\right)\right]. \quad (17)$$

where $\lambda_i = 2\Delta_i - \Delta_1 - \Delta_2 - \Delta_3$, $i = 1,2,3$ (Δ_i is the dimension of i-th field in the correlation function), Θ is the "superprojective invariant" of three points

$$\Theta = (z_{12}z_{13}z_{23})^{-1/2}\left(z_{23}\theta_1 + z_{31}\theta_2 + z_{12}\theta_3 - \frac{1}{2}\theta_1\theta_2\theta_3\right). \quad (18)$$

Supersymmetry allows one to express coefficients $\tilde{C}^{\epsilon}_{[\alpha_1],[\alpha_2],(\alpha_3)}$ and $d^{\epsilon}_{[\alpha_1],[\alpha_2],(\alpha_3)}$ via $C^{\epsilon}_{[\alpha_1],[\alpha_2],(\alpha_3)}$ as follows

$$\tilde{C}^{\epsilon}_{[\alpha_1],[\alpha_2],(\alpha_3)} = i\epsilon\left[\left(\beta_1^2+\beta_2^2\right)C^{\epsilon}_{[\alpha_1],[\alpha_2],(\alpha_3)} - 2\beta_1\beta_2 C^{-\epsilon}_{[\alpha_1],[\alpha_2],(\alpha_3)}\right], \quad (19)$$

$$d^{\epsilon}_{[\alpha_1],[\alpha_2],(\alpha_3)} = ie^{-\frac{i\pi\epsilon}{4}}\left[\beta_2 C^{\epsilon}_{[\alpha_1],[\alpha_2],(\alpha_3)} - \beta_1 C^{-\epsilon}_{[\alpha_1],[\alpha_2],(\alpha_3)}\right]. \quad (20)$$

The numerical coefficients $C, \tilde{C}, d, \tilde{d}$ are called structure constants and their calculation is the main purpose of this work.

3 Degenerated Fields and Four Point Correlation Functions

For some special values of parameter α the primary fields ϕ_α (R^{ϵ}_{α}) become degenerated. This means, that corresponding module Verma, i.e. the space, obtained with the help of

successive actions by operators S_k,L_n, $\bar{S}_k$, $\bar{L_n}$ with $k,n<0$ on primary fields ϕ_α, (R^ϵ_α), contains "null vector", i.e. some field $\chi_{\Delta+L}$ with properties

$$L_n\chi_{\Delta+L}=S_k\chi_{\Delta+L}=0,\ \text{for}\ n,k>0,\ L_0\chi_{\Delta+L}=(\Delta+L)\chi_{\Delta+L}, \tag{21}$$

where Δ is the conformal dimension of the field ϕ_α (R^ϵ_α) and the level of degeneracy L is some integer or half integer. To obtain an irreducible module, one has to factorize the Verma modul over all submodules, generated by the null vectors. In the field theory language this means, that we must put

$$\chi_{\Delta+L}=0. \tag{22}$$

Let us consider an example of degeneration, which plays an important role in further discussion. The Ramond field R^ϵ_α is degenerated at the level $L=1$, if $\alpha=-b/2$, or $\alpha=-1/2b$. The corresponding null vector has the form

$$\chi=(\kappa L_{-1}-S_{-1}S_0)R^\epsilon_\alpha=0, \tag{23}$$

where

$$\kappa=\begin{cases} 1+\dfrac{1}{2b^2}, & \text{if}\quad \alpha=-\dfrac{b}{2}, \\ \\ 1+\dfrac{2b^2}{2}, & \text{if}\quad \alpha=-\dfrac{1}{2b}. \end{cases} \tag{24}$$

Below we'll mainly consider the case $\alpha=-b/2$. To find corresponding formulae for the case $\alpha=-1/2b$ one simply has to replace $b\leftrightarrow 1/b$. Following to [17], it is easy to show, that the correlation function $\langle\Phi_{\alpha_3}\Phi_{\alpha_2}R^\epsilon_\alpha R^{\epsilon_1}_{\alpha_1}\rangle$, as a consequence of eq. (22), satisfies the following differential equation:

$$\begin{aligned}
&\kappa\frac{\partial}{\partial x_1}\langle\Phi_{\alpha_3}\left(Z_3,\bar{Z}_3\right)\Phi_{\alpha_2}\left(Z_2,\bar{Z}_2\right)R^\epsilon_\alpha\left(z,\bar{z}\right)R^{\epsilon_1}_{\alpha_1}\left(z_1,\bar{z}_1\right)\rangle= \\
&=\sum_{i=2}^{3}\sqrt{\frac{(z_i-z)(z_i-z_1)}{z-z_1}}\left\{-\frac{2\Delta_{\alpha_i}\theta_i}{(z-z_i)^2}+\frac{1}{z-z_i}\hat{Q}_i\right\}\times \\
&\times\langle\Phi_{\alpha_3}\left(Z_3,\bar{Z}_3\right)\Phi_{\alpha_2}\left(Z_2,\bar{Z}_2\right)S_0R^\epsilon_\alpha\left(z,\bar{z}\right)R^{\epsilon_1}_{\alpha_1}\left(z_1,\bar{z}_1\right)\rangle+ \\
&+\frac{\beta^2}{2(z-z_1)}\langle\Phi_{\alpha_3}\left(Z_3,\bar{Z}_3\right)\Phi_{\alpha_2}\left(Z_2,\bar{Z}_2\right)R^\epsilon_\alpha\left(z,\bar{z}\right)R^{\epsilon_1}_{\alpha_1}\left(z_1,\bar{z}_1\right)\rangle- \\
&-\frac{i\epsilon}{z-z_1}\langle\Phi_{\alpha_3}\left(Z_3,\bar{Z}_3\right)\Phi_{\alpha_2}\left(Z_2,\bar{Z}_2\right)S_0R^\epsilon_\alpha\left(z,\bar{z}\right)S_0R^{\epsilon_1}_{\alpha_1}\left(z_1,\bar{z}_1\right)\rangle,
\end{aligned} \tag{25}$$

where $\hat{Q}_i = \partial/\partial\theta_i - \theta_i\partial/\partial z_i$, two possible choices of α and corresponding κ are given by the eq. (24) and β can be expressed via α with the help of eq.(13). Equation (25) (and the analogous differential equation over complex conjugate variables) makes it possible to obtain correlation function $\langle\Phi_{\alpha_3}\Phi_{\alpha_2}R^{\epsilon}_{\alpha}R^{\epsilon_1}_{\alpha_1}\rangle$. In fact, due to superconformal symmetry it's quite sufficient to obtain correlation functions

$$G^{\epsilon}(z,\bar{z}) = \lim_{|z_3\bar{z}_3|\to\infty} |z_3\bar{z}_3|^{2\Delta_{\alpha_3}} \langle\phi_{\alpha_3}(z_3,\bar{z}_3)\,\phi_{\alpha_2}(1)\,R^{\epsilon}_{\alpha}(z,\bar{z})\,R^{\epsilon}_{\alpha_1}(0)\rangle \tag{26}$$

and

$$H^{\epsilon}(z,\bar{z}) = \lim_{|z_3\bar{z}_3|\to\infty} |z_3\bar{z}_3|^{2\Delta_{\alpha_3}} \langle\phi_{\alpha_3}(z_3,\bar{z}_3)\,\psi_{\alpha_2}(1)\,S_0 R^{\epsilon}_{\alpha}(z,\bar{z})\,R^{\epsilon}_{\alpha_1}(0)\rangle, \tag{27}$$

Rewriting (25) in component language and adjusting coordinates appropriately we obtain

$$\left(\kappa\frac{\partial}{\partial z} - \frac{\beta^2}{2z}\right)G^{\epsilon}(z,\bar{z}) = \frac{\beta\beta_1}{z}\quad G^{-\epsilon}(z,\bar{z}) - \frac{1}{\sqrt{z(1-z)}}H^{\epsilon}(z,\bar{z}), \tag{28}$$

$$\left(\kappa\frac{\partial}{\partial z} - \frac{\beta^2}{2z}\right)H^{\epsilon}(z,\bar{z}) = -\frac{\beta\beta_1}{z}\quad H^{-\epsilon}(z,\bar{z}) + \frac{2\Delta_{\alpha_2}\beta^2}{(1-z)\sqrt{z(1-z)}}G^{\epsilon}(z,\bar{z}) -$$
$$-\frac{\beta^2}{\sqrt{z(1-z)}}\left(\gamma - z\frac{\partial}{\partial z}\right)G^{\epsilon}(z,\bar{z}), \tag{29}$$

where $\gamma = \Delta_{\alpha_3} - \Delta_{\alpha_2} - \Delta_{[\alpha]} - \Delta_{[\alpha_1]}$, $\beta_1 = (Q/2-\alpha_1)/\sqrt{2}$. It's convenient to introduce new functions $G_{\epsilon}(z,\bar{z})$ and $H_{\epsilon}(z,\bar{z})$ as follows:

$$G^{\epsilon}(z,\bar{z}) = G_{+}(z,\bar{z}) + \epsilon G_{-}(z,\bar{z}), \tag{30}$$

$$H^{\epsilon}(z,\bar{z}) = H_{+}(z,\bar{z}) + \epsilon H_{-}(z,\bar{z}), \tag{31}$$

The functions $G_{\epsilon}(z,\bar{z})$ and $H_{\epsilon}(z,\bar{z})$ obey the following system of differential equations:

$$\left(\kappa\frac{\partial}{\partial z} - \frac{\beta^2}{2z}\right)G_{\epsilon}(z,\bar{z}) = \frac{\beta\beta_1\epsilon}{z}\quad G_{\epsilon}(z,\bar{z}) - \frac{1}{\sqrt{z(1-z)}}H_{\epsilon}(z,\bar{z}), \tag{32}$$

$$\left(\kappa\frac{\partial}{\partial z} - \frac{\beta^2}{2z}\right)H_{\epsilon}(z,\bar{z}) = \tag{33}$$
$$= -\frac{\beta\beta_1\epsilon}{z}H_{\epsilon}(z,\bar{z}) - \frac{\beta^2}{\sqrt{z(1-z)}}\left(\gamma - \frac{2\Delta_{\alpha_2}}{1-z} - z\frac{\partial}{\partial z}\right)G_{\epsilon}(z,\bar{z}).$$

Now, it's not difficult to exclude $H_\epsilon(z,\bar{z})$ and obtain the following second order linear differential equation (in fact it coincides with Gauss hypergeometric equation):

$$z(1-z)\frac{\partial^2}{\partial z^2}U_\epsilon(z,\bar{z})-\left[c_\epsilon-(a_\epsilon+b_\epsilon+1)z\right]\frac{\partial}{\partial z}U_\epsilon(z,\bar{z})-a_\epsilon b_\epsilon U_\epsilon(z,\bar{z})=0, \tag{34}$$

where

$$G_\epsilon(z,\bar{z})=(z\bar{z})^{\alpha_\epsilon}\left[(1-z)(1-\bar{z})\right]^{\beta_\epsilon}U_\epsilon(z,\bar{z}), \tag{35}$$

$$\begin{aligned}
\alpha_\epsilon &= \frac{1}{4}\left(\frac{1}{2}+b^2+b\epsilon(Q-2\alpha_1)\right); \quad \beta_\epsilon=\frac{1}{4}\left(1+b^2+b\epsilon(Q-2\alpha_2)\right); \\
a_\epsilon &= \frac{1}{4}\left(1+b\epsilon(3Q-2\alpha_1-2\alpha_2-2\alpha_3)\right); \\
b_\epsilon &= \frac{1}{4}\left(1+b\epsilon(Q-2\alpha_1-2\alpha_2+2\alpha_3)\right); \\
c_\epsilon &= \frac{1}{2}\left(1+b\epsilon(Q-2\alpha_1)\right).
\end{aligned} \tag{36}$$

Taking into account the mutual locality of the fields R^ϵ_α, $R^\epsilon_{\alpha_1}$, ϕ_{α_1}, ϕ_{α_2} and that $U_\epsilon(z,\bar{z})$ obeys the same differential equation also over the variable $\bar{z}$ it is straightforward to obtain following expression:

$$\begin{aligned}
G_\epsilon(z,\bar{z}) &= (z\bar{z})^{\alpha_\epsilon}\left[(1-z)(1-\bar{z})\right]^{\beta_\epsilon}\times \\
&\times\left\{g_\epsilon|F(a_\epsilon,b_\epsilon,c_\epsilon,z)|^2+\tilde{g}_\epsilon(z\bar{z})^{1-c_\epsilon}|F(1+a_\epsilon-c_\epsilon,1+b_\epsilon-c_\epsilon,2-c_\epsilon,z)|^2\right\},
\end{aligned} \tag{37}$$

where

$$\tilde{g}_\epsilon=-g_\epsilon\frac{\Gamma^2(c_\epsilon)\gamma(1-a_\epsilon)\gamma(1-b_\epsilon)}{\Gamma^2(2-c_\epsilon)\gamma(c_\epsilon-a_\epsilon)\gamma(c_\epsilon-b_\epsilon)}, \tag{38}$$

and g_ϵ are some constants to be defined later. For our purposes it is useful also the following equivalent expression for $G_\epsilon(z,\bar{z})$, which makes $z\to 1$ asymptotics of the correlation function $G_\epsilon(z,\bar{z})$ transparent:

$$\begin{aligned}
G_\epsilon(z,\bar{z}) &= (z\bar{z})^{\alpha_\epsilon}\left[(1-z)(1-\bar{z})\right]^{\beta_\epsilon}\left\{f_\epsilon|F(a_\epsilon,b_\epsilon,a_\epsilon+b_\epsilon+1-c_\epsilon,1-z)|^2+\right. \\
&\left.+\tilde{f}_\epsilon\left((1-z)(1-\bar{z})\right)^{c_\epsilon-a_\epsilon-b_\epsilon}|F(c_\epsilon-a_\epsilon,c_\epsilon-b_\epsilon,c_\epsilon+1-a_\epsilon-b_\epsilon,1-z)|^2\right\},
\end{aligned} \tag{39}$$

where

$$f_\epsilon=g_\epsilon\frac{\gamma(c_\epsilon)\gamma(c_\epsilon-a_\epsilon-b_\epsilon)}{\gamma(c_\epsilon-a_\epsilon)\gamma(c_\epsilon-b_\epsilon)}; \quad \tilde{f}_\epsilon=g_\epsilon\frac{\gamma(c_\epsilon)\gamma(a_\epsilon+b_\epsilon-c_\epsilon)}{\gamma(a_\epsilon)\gamma(b_\epsilon)}. \tag{40}$$

4 Three Point Functions and Reflection Amplitudes

Expressions (37) and (39) show, that operator product expansion of the field $R^{\epsilon}_{-b/2}$ with arbitrary other primary field (from the NS or R-sector) contains only finite number of primary fields (see eq.(42)). In fact, this property holds for all degenerated primary fields, which are characterized by the following spectrum of parameter α:

$$\alpha_{n,m} = \frac{1-n}{2}b + \frac{1-m}{2b}, \tag{41}$$

where n, m are positive integers and $n-m = 0(mod2)$ ($n-m = 1(mod2)$), if the corresponding field is from NS-sector (R-sector). The above mentioned property makes it possible (and convenient) to choose usual in CFT discrete unit normalization for degenerated fields instead of continuous normalization (14), (15) . Investigating $z \to 0$ and $z \to 1$ singularities of correlation functions (37), (39) and identifying intermediate states (see [17]) we see that the following operator product expansions are valid:

$$\begin{aligned} R^{\epsilon}_{-b/2}(z,\bar{z})R^{\epsilon}_{\alpha_1}(0) &= \sum_{\sigma=\pm 1}(z\bar{z})^{\Delta_{\alpha_1+\sigma b/2}-\Delta_{[-b/2]}-\Delta_{[\alpha_1]}}\Big(C^{\epsilon,(\alpha_1+\sigma b/2)}_{[-b/2],[\alpha_1]}\phi_{\alpha_1+\sigma b/2}(0)+ \\ &+ \quad |z|(2\Delta_{\alpha_1+\sigma b/2})^{-2}\tilde{C}^{\epsilon,(\alpha_1+\sigma b/2)}_{[-b/2],[\alpha_1]}\tilde{\phi}_{\alpha_1+\sigma b/2}(0)+\ldots\Big) \\ R^{\epsilon}_{-b/2}(z,\bar{z})\phi_{\alpha_2}(0) &= \sum_{\sigma=\pm 1}(z\bar{z})^{\Delta_{[\alpha_2+\sigma b/2]}-\Delta_{[-b/2]}-\Delta_{\alpha_2}}\Big(C^{\epsilon,([\alpha_2+\sigma b/2]}_{[-b/2],(\alpha_2)}R^{\epsilon}_{[\alpha_2+\sigma b/2]}(0)+\ldots\Big). \end{aligned} \tag{42}$$

Let me note that owing to self-consistent normalization in NS- and R-sectors (14,15) $C^{\epsilon,(\alpha_1\pm b/2)}_{[-b/2],[\alpha]} = C^{\epsilon,[\alpha]}_{[-b/2],(\alpha\pm b/2)}$. Taking into account (42) it is easy to connect the constants $g_\epsilon, \tilde{g}_\epsilon, f_\epsilon, \tilde{f}_\epsilon$ from (37,39) with the structure constants:

$$\begin{aligned} & g_+ = C^{\epsilon,(\alpha_1+b/2)}_{[-b/2],[\alpha_1]}C_{(\alpha_1+b/2),(\alpha_2),(\alpha_3)}; \\ & -\Big(2\Delta_{\alpha_1-b/2}\Big)^2\tilde{g}_+ = \tilde{C}^{\epsilon,(\alpha_1-b/2)}_{[-b/2],[\alpha_1]}\tilde{C}_{(\alpha_1-b/2),(\alpha_2),(\alpha_3)}; \\ & \epsilon g_- = C^{\epsilon,(\alpha_1-b/2)}_{[-b/2],[\alpha_1]}C_{(\alpha_1-b/2),(\alpha_2),(\alpha_3)}; \\ & -\epsilon\Big(2\Delta_{\alpha_1+b/2}\Big)^2\tilde{g}_- = \tilde{C}^{\epsilon,(\alpha_1+b/2)}_{[-b/2],[\alpha_1]}\tilde{C}_{(\alpha_1+b/2),(\alpha_2),(\alpha_3)}; \\ & f_+ + \epsilon\tilde{f}_- = C^{\epsilon,[\alpha_2+b/2]}_{[-b/2],(\alpha_2)}C^{\epsilon}_{[\alpha_1],[\alpha_2+b/2],(\alpha_3)}; \\ & \tilde{f}_+ + \epsilon f_- = C^{\epsilon,[\alpha_2-b/2]}_{[-b/2],(\alpha_2)}C^{\epsilon}_{[\alpha_1],[\alpha_2-b/2],(\alpha_3)}. \end{aligned} \tag{43}$$

As the quantities g_ϵ, $\tilde{g}_\epsilon$, f_ϵ, $\tilde{f}_\epsilon$ are connected via eqs.(38), (40), the relations (43) are, in fact, highly non trivial functional relations for structure constants. Before solving this functional relations, let us consider a special case with $\alpha_1 = -b/2$. As the field ϕ_0, appearing in s-channel, is the unit operator, in this case we have to put $g_+ = \Delta(\alpha_2 - \alpha_3)$, $g_- = f_- = \tilde{f}_- = 0$ in (43). In this case it follows from (43), that

$$C^{\epsilon,[\alpha+b/2]}_{[-b/2],(\alpha)} = \epsilon \left[\frac{\gamma(Qb)\gamma(\frac{(2\alpha-Q)b}{2})}{\gamma(\frac{Qb}{2})\gamma(\alpha b)}\right]^{1/2},$$

$$C^{\epsilon,[\alpha-b/2]}_{[-b/2],(\alpha)} = \left[\frac{\gamma(Qb)\gamma(\frac{(Q-2\alpha)b}{2})}{\gamma(\frac{Qb}{2})\gamma((Q-\alpha)b)}\right]^{1/2}. \tag{44}$$

Using (19), we easily obtain also the following structure constants:

$$\tilde{C}^{\epsilon,(\alpha-b/2)}_{[-b/2],[\alpha]} = \frac{i}{2}(Q-\alpha+\frac{b}{2})^2 \left[\frac{\gamma(Qb)\gamma(\alpha b+\frac{1}{2}-Qb)}{\gamma\left(\frac{Qb}{2}\right)\gamma\left(\alpha b+\frac{1}{2}-\frac{Qb}{2}\right)}\right]^{1/2},$$

$$\tilde{C}^{\epsilon,(\alpha+b/2)}_{[-b/2],[\alpha]} = \frac{i\epsilon}{2}(\alpha+\frac{b}{2})^2 \left[\frac{\gamma(Qb)\gamma(\alpha b+\frac{1}{2}-\frac{Qb}{2})}{\gamma\left(\frac{Qb}{2}\right)\gamma\left(\alpha b+\frac{1}{2}\right)}\right]^{1/2}. \tag{45}$$

Inserting (44), (45) into (43), excluding g_+, $\tilde{g_+}$ g_-, $\tilde{g_-}$ and $\tilde{C}_{(\alpha_1\pm b/2),(\alpha_2),(\alpha_3)}$, we obtain the following functional relation for $C_{(\alpha_1),(\alpha_2),(\alpha_3)}$:

$$\begin{aligned}\frac{C_{(\alpha_1+2b),(\alpha_2),(\alpha_3)}}{C_{(\alpha_1),(\alpha_2),(\alpha_3)}} &= b^4\left(\frac{\gamma\left(\alpha_1 b-\frac{Qb}{2}+2b^2\right)\gamma\left(\frac{2\alpha_1-Q}{2b}+3\right)}{\gamma\left(\alpha_1 b-\frac{Qb}{2}\right)\gamma\left(\frac{2\alpha_1-Q}{2b}+1\right)}\right)^{1/2}\times\\ &\times\gamma\left(\alpha_1 b-\frac{Qb}{2}\right)\gamma\left(\alpha_1 b+\frac{Qb}{2}-1\right)\gamma\left(\alpha_1 b-Qb+1\right)\gamma\left(\alpha_1 b\right)\times\\ &\times\frac{\gamma\left(\frac{\delta_1 b}{2}-Qb+1\right)\gamma\left(\frac{\delta_1-Q}{2}b+1\right)}{\gamma\left(\frac{\delta b}{2}\right)\gamma\left(\frac{\delta-Q}{2}b\right)\gamma\left(\frac{\delta_2+Q}{2}b\right)\gamma\left(\frac{\delta_2}{2}b\right)\gamma\left(\frac{\delta_3+Q}{2}b\right)\gamma\left(\frac{\delta_3}{2}b\right)},\end{aligned} \tag{46}$$

where $\delta = \alpha_1+\alpha_2+\alpha_3$, $\delta_i = \delta - 2\alpha_i$. An analogous functional relation, obtained via the substitution $b \leftrightarrow 1/b$, is also valid

$$\begin{aligned}\frac{C_{(\alpha_1+2/b),(\alpha_2),(\alpha_3)}}{C_{(\alpha_1),(\alpha_2),(\alpha_3)}} &= b^{-4}\left(\frac{\gamma\left(\frac{2\alpha_1-Q}{2b}+\frac{2}{b^2}\right)\gamma\left(\alpha_1 b-\frac{Qb}{2}+3\right)}{\gamma\left(\frac{2\alpha_1-Q}{2b}\right)\gamma\left(\alpha_1 b-\frac{Qb}{2}+1\right)}\right)^{1/2}\times\\ &\times\gamma\left(\frac{2\alpha_1-Q}{2b}\right)\gamma\left(\frac{2\alpha_1+Q}{2b}-1\right)\gamma\left(\frac{\alpha_1-Q}{b}+1\right)\gamma\left(\frac{\alpha_1}{b}\right)\times\end{aligned} \tag{47}$$

$$\times \frac{\gamma\left(\frac{\delta_1-2Q}{2b}+1\right)\gamma\left(\frac{\delta_1-Q}{2b}+1\right)}{\gamma\left(\frac{\delta}{2b}\right)\gamma\left(\frac{\delta-Q}{2b}\right)\gamma\left(\frac{\delta_2+Q}{2b}\right)\gamma\left(\frac{\delta_2}{2b}\right)\gamma\left(\frac{\delta_3+Q}{2b}\right)\gamma\left(\frac{\delta_3}{2b}\right)},$$

Functional relations (46), (47) (and the condition, that $C_{(\alpha_1),(\alpha_2),(\alpha_3)}$ is a symmetric function of α_1, α_2 and α_3) determine the structure constant $C_{(\alpha_1),(\alpha_2),(\alpha_3)}$ up to an overall constant factor. The last statement follows from the fact, that the ratio of any two solutions of (46),(47) would be periodic with two real (in general incommensurable, if b^2 is irrational) periods b and $1/b$. But such a function (under some natural assumptions of general character) must be constant.

The solution of (46), (47) can be expressed via the function $\Upsilon(x, Q)$, introduced by A.B. and Al.B.Zamolodchikovs in [16]

$$\log \Upsilon(x, Q) = \int_0^\infty \frac{dt}{t}\left[\left(\frac{Q}{2}-x\right)^2 e^{-t} - \frac{\sinh^2\left(\frac{Q}{2}-x\right)\frac{t}{2}}{\sinh\frac{bt}{2}\sinh\frac{t}{2b}}\right]. \tag{48}$$

Using the properties of $\Upsilon(x)$ (here and below we suppress the parameter Q) [16]

$$\Upsilon(x+b) = \gamma(bx) b^{1-2bx}\Upsilon(x), \tag{49}$$

$$\Upsilon(x+1/b) = \gamma(x/b) b^{2x/b-1}\Upsilon(x), \tag{50}$$

one can easily check, that the following expression is the solution of (46),(47):

$$C_{(\alpha_1),(\alpha_2),(\alpha_3)} = C_0\Upsilon_0 b^{(3Q-\alpha_1-\alpha_2-\alpha_3)(1/b-b)} \times$$
$$\times \prod_{i=1}^{3}\left[\left(\gamma\left(1+\frac{Q}{2b}-\frac{\alpha_i}{b}\right)\gamma\left(\frac{Qb}{2}-\alpha_i b\right)\right)^{1/2}\prod_{\sigma\in 0,1}\frac{\Upsilon\left(\alpha_i+\frac{\sigma Q}{2}\right)}{\Upsilon\left(\frac{\delta_i+\sigma Q}{2}\right)\Upsilon\left(\frac{\delta-\sigma Q}{2}\right)}\right]. \tag{51}$$

where C_0 is a constant, whose value depends on concrete choice of normalizing function $\Delta(\alpha)$ in (14) and

$$\Upsilon_0 = \frac{d\Upsilon(x)}{dx}|_{x=0}. \tag{52}$$

Using (43) and (51), for the structure constant $\tilde{C}_{(\alpha_1),(\alpha_2),(\alpha_3)}$ we obtain

$$\begin{aligned}\tilde{C}_{(\alpha_1),(\alpha_2),(\alpha_3)} &= -2iC_0\Upsilon_0 b^{(3Q-\alpha_1-\alpha_2-\alpha_3)(1/b-b)} \times \\ &\times \prod_{i=1}^{3}\left[\left(\gamma\left(1+\frac{Q}{2b}-\frac{\alpha_i}{b}\right)\gamma\left(\frac{Qb}{2}-\alpha_i b\right)\right)^{1/2} \times \right. \\ &\times \left.\frac{\Upsilon(\alpha_i)\Upsilon\left(\alpha_i+\frac{Q}{2}\right)}{\Upsilon\left(\frac{\delta_i}{2}+\frac{1}{2b}\right)\Upsilon\left(\frac{\delta_i}{2}+\frac{b}{2}\right)\Upsilon\left(\frac{\delta}{2}-\frac{1}{2b}\right)\Upsilon\left(\frac{\delta}{2}-\frac{b}{2}\right)}\right].\end{aligned} \tag{53}$$

Now, it is not difficult to find corresponding expressions for the Ramond sector. Indeed, using (40), (43) and (44) we get

$$f_{+}=\left[\frac{\gamma\left(Qb\right)\gamma\left(\frac{Q-2\alpha_1}{2}b+\frac{1}{2}\right)}{\gamma\left(\frac{Qb}{2}\right)\gamma\left(\alpha_1 b+\frac{1}{2}\right)}\right]^{1/2}\frac{\gamma\left(\frac{2\alpha_2-Q}{2}b\right)C_{(\alpha_1+b/2),(\alpha_2),(\alpha_3)}}{\gamma\left(\frac{2\delta_1-Q}{4}b+\frac{1}{4}\right)\gamma\left(\frac{Q-2\delta_2}{4}b+\frac{1}{4}\right)}, \tag{54}$$

$$f_{-}=\left[\frac{\gamma\left(Qb\right)\gamma\left((\alpha_1-Q)b+\frac{1}{2}\right)}{\gamma\left(\frac{Qb}{2}\right)\gamma\left(\frac{Q-2\alpha_1}{2}b+\frac{1}{2}\right)}\right]^{1/2}\frac{\gamma\left(\frac{Q-2\alpha_2}{2}b\right)C_{(\alpha_1-b/2),(\alpha_2),(\alpha_3)}}{\gamma\left(\frac{Q-2\delta_1}{4}b+\frac{1}{4}\right)\gamma\left(\frac{2\delta_2-Q}{4}b+\frac{1}{4}\right)}. \tag{55}$$

Inserting (54), (55) into (43) and shifting the parameter $\alpha_2\to\alpha_2-b/2$, we obtain:

$$\begin{aligned}
&C^{\epsilon}_{[\alpha_1],[\alpha_2],(\alpha_3)}=\\
&=C_0\Upsilon_0 b^{(3Q-\delta)(1/b-b)-2}\left(\frac{\gamma\left(\frac{Qb}{2}-\alpha_3 b\right)\gamma\left(1+\frac{Q}{2b}-\frac{\alpha_3}{b}\right)}{\gamma\left(\alpha_1 b-\frac{b^2}{2}\right)\gamma\left(\alpha_2 b-\frac{b^2}{2}\right)\gamma\left(\frac{\alpha_1}{b}-\frac{1}{2b^2}\right)\gamma\left(\frac{\alpha_2}{b}-\frac{1}{2b^2}\right)}\right)^{1/2}\times\\
&\times\left[\frac{\Upsilon\left(\alpha_1+\frac{b}{2}\right)\Upsilon\left(\alpha_2+\frac{b}{2}\right)\Upsilon\left(\alpha_1+\frac{1}{2b}\right)\Upsilon\left(\alpha_2+\frac{1}{2b}\right)\Upsilon\left(\alpha_3\right)\Upsilon\left(\alpha_3+\frac{Q}{2}\right)}{\Upsilon\left(\frac{\delta_1}{2}\right)\Upsilon\left(\frac{\delta_1+Q}{2}\right)\Upsilon\left(\frac{\delta_2}{2}\right)\Upsilon\left(\frac{\delta_2+Q}{2}\right)\Upsilon\left(\frac{\delta_3+b}{2}\right)\Upsilon\left(\frac{b\delta_3+1}{2b}\right)\Upsilon\left(\frac{\delta-b}{2}\right)\Upsilon\left(\frac{b\delta-1}{2b}\right)}+\right.\\
&\left.+\frac{\epsilon\Upsilon\left(\alpha_1+\frac{b}{2}\right)\Upsilon\left(\alpha_2+\frac{b}{2}\right)\Upsilon\left(\alpha_1+\frac{1}{2b}\right)\Upsilon\left(\alpha_2+\frac{1}{2b}\right)\Upsilon\left(\alpha_3\right)\Upsilon\left(\alpha_3+\frac{Q}{2}\right)}{\Upsilon\left(\frac{\delta_1+b}{2}\right)\Upsilon\left(\frac{\delta_2+b}{2}\right)\Upsilon\left(\frac{b\delta_1+1}{2b}\right)\Upsilon\left(\frac{b\delta_2+1}{2b}\right)\Upsilon\left(\frac{\delta_3}{2}\right)\Upsilon\left(\frac{\delta_3+Q}{2}\right)\Upsilon\left(\frac{\delta}{2}\right)\Upsilon\left(\frac{\delta-Q}{2}\right)}\right].
\end{aligned} \tag{56}$$

Up to now we have assumed all α's to be restricted to $\alpha\leq Q/2$, but the expressions (51), (53), (56) are defined out of this region as well and it is interesting to note, that the following relations are valid:

$$C_{(\alpha_1),(\alpha_2),(\alpha_3)}=C_{(Q-\alpha_1),(\alpha_2),(\alpha_3)}, \tag{57}$$

$$\tilde{C}_{(\alpha_1),(\alpha_2),(\alpha_3)}=\tilde{C}_{(Q-\alpha_1),(\alpha_2),(\alpha_3)}, \tag{58}$$

$$C^{\epsilon}_{[\alpha_1],[\alpha_2],(\alpha_3)}=\epsilon C^{\epsilon}_{[Q-\alpha_1],[\alpha_2],(\alpha_3)}=C^{\epsilon}_{[\alpha_1],[\alpha_2],(Q-\alpha_3)}. \tag{59}$$

Let us introduce normalizing coefficients in (4), (6) explicitly

$$\exp\alpha\Phi_{SL}=\nu(\alpha)\Phi_{\alpha}. \tag{60}$$

$$\sigma^{(\epsilon)}\exp\alpha\phi_{SL}=\rho(\alpha)R^{\epsilon}_{\alpha} \tag{61}$$

Similar to ordinary bosonic Liouville theory, the correlation functions of exponential fields have poles if total charge $\sum\alpha_i=Q-nb$; $n\in 0,1,2,\ldots$. The residues of these poles can

be calculated via perturbative expansion over cosmological constant μ (in fact only the term $\sim \mu^n$ contributes). In particular, if $n=0$ we have correlation functions in a free theory ($\mu=0$) with background charge Q at infinity. So, for the three point functions we have:

$$
\begin{aligned}
&\mathrm{res}_{\sum\alpha_i=Q}\langle e^{\alpha_1\Phi_{SL}}e^{\alpha_2\Phi_{SL}}e^{\alpha_3\Phi_{SL}}\rangle = \langle e^{\alpha_1\Phi_{SL}}e^{\alpha_2\Phi_{SL}}e^{(Q-\alpha_1-\alpha_2)\Phi_{SL}}\rangle_{\mu=0} = \\
&= |Z_{12}|^{2\lambda_3}|Z_{13}|^{2\lambda_2}|Z_{23}|^{2\lambda_1},
\end{aligned}
\tag{62}
$$

$$
\begin{aligned}
&\mathrm{res}_{\sum\alpha_i=Q}\langle e^{\alpha_1\phi_{SL}}\sigma^{\epsilon}e^{\alpha_2\phi_{SL}}\sigma^{\epsilon}e^{\alpha_3\phi_{SL}}\rangle = \langle e^{\alpha_1\phi_{SL}}\sigma^{\epsilon}e^{\alpha_2\phi_{SL}}\sigma^{\epsilon}e^{(Q-\alpha_1-\alpha_2)\phi_{SL}}\rangle_{\mu=0} = \\
&= |z_{12}|^{2\lambda_3}|z_{13}|^{2\lambda_2}|z_{23}|^{2\lambda_1},
\end{aligned}
\tag{63}
$$

The relations (62), (63) determine normalizing functions $\nu(\alpha)$, $\rho(\alpha)$ up to constant parameters κ, C_0:

$$
\nu(\alpha) = \left(2b^{2Q(1/b-b)}C_0\right)^{-1/3}\kappa^{\alpha-\frac{Q}{3}}\left(\frac{\gamma\left(\frac{\alpha}{b}-\frac{Q}{2b}\right)}{\gamma\left(-\alpha b+\frac{Qb}{2}\right)}\right)^{1/2}, \tag{64}
$$

$$
\rho(\alpha) = \left(2b^{2Q(1/b-b)}C_0\right)^{-1/3}\kappa^{\alpha-\frac{Q}{3}}\left(\frac{b^2\gamma\left(\frac{\alpha}{b}-\frac{1}{2b^2}\right)}{\gamma\left(1-\alpha b+\frac{b^2}{2}\right)}\right)^{1/2}. \tag{65}
$$

To express unknown constant κ, via cosmological constant μ and coupling constant b, one can in a similar way investigate the case $n=1$:

$$
\begin{aligned}
&\mathrm{res}_{\sum\alpha_i=1/b}\langle e^{\alpha_1\Phi_{SL}}e^{\alpha_2\Phi_{SL}}e^{\alpha_3\Phi_{SL}}\rangle = \\
&= i\mu\int d^2z_4d^2\theta_4\langle e^{\alpha_1\Phi_{SL}}e^{\alpha_2\Phi_{SL}}e^{\left(\frac{1}{b}-\alpha_1-\alpha_2\right)\Phi_{SL}}e^{b\Phi_{SL}\left(Z_4,\bar{Z}_4\right)}\rangle_{\mu=0} = \\
&= -i\pi\mu\frac{\gamma\left(b\alpha_1+b\alpha_2\right)}{\gamma\left(b\alpha_1\right)\gamma\left(b\alpha_2\right)}\Theta\bar{\Theta}|Z_{12}|^{2\lambda_3}|Z_{13}|^{2\lambda_2}|Z_{23}|^{2\lambda_1}.
\end{aligned}
\tag{66}
$$

Using eq. (66) we easily obtain, that

$$
\kappa = b^{b-1/b}\left(\frac{\pi\mu}{2}\gamma\left(\frac{1+b^2}{2}\right)\right)^{-1/b}. \tag{67}
$$

From (51), (56), (64), (65), (67) we obtain final expressions for the structure constants of exponential fields $\exp\alpha\phi_{SL}$, $\sigma^{(\epsilon)}\exp\alpha\phi_{SL}$:

$$
\mathbf{C}_{(\alpha_1),(\alpha_2),(\alpha_3)} = \frac{1}{2}\left(\frac{\pi\mu}{2}\gamma\left(\frac{1+b^2}{2}\right)b^{2-2b^2}\right)^{\frac{Q-\delta}{b}}\times
$$

$$\times\Upsilon_0\prod_{i=1}^{3}\prod_{\sigma\in 0,1}\frac{\Upsilon\left(\alpha_i+\frac{\sigma Q}{2}\right)}{\Upsilon\left(\frac{\delta_i+\sigma Q}{2}\right)\Upsilon\left(\frac{\delta-\sigma Q}{2}\right)}, \tag{68}$$

$$\tilde{\mathbf{C}}_{(\alpha_1),(\alpha_2),(\alpha_3)}=-i\left(\frac{\pi\mu}{2}\gamma\left(\frac{1+b^2}{2}\right)b^{2-2b^2}\right)^{\frac{Q-\delta}{b}}\Upsilon_0\times$$
$$\times\prod_{i=1}^{3}\frac{\Upsilon\left(\alpha_i\right)\Upsilon\left(\alpha_i+\frac{Q}{2}\right)}{\Upsilon\left(\frac{\delta_i}{2}+\frac{1}{2b}\right)\Upsilon\left(\frac{\delta_i}{2}+\frac{b}{2}\right)\Upsilon\left(\frac{\delta}{2}-\frac{1}{2b}\right)\Upsilon\left(\frac{\delta}{2}-\frac{b}{2}\right)}, \tag{69}$$

$$\mathbf{C}^{\epsilon}_{[\alpha_1],[\alpha_2],(\alpha_3)}=\frac{1}{2}\left(\frac{\pi\mu}{2}\gamma\left(\frac{1+b^2}{2}\right)b^{2-2b^2}\right)^{\frac{Q-\delta}{b}}\Upsilon_0\times$$
$$\times\left[\frac{\Upsilon\left(\alpha_1+\frac{b}{2}\right)\Upsilon\left(\alpha_2+\frac{b}{2}\right)\Upsilon\left(\alpha_1+\frac{1}{2b}\right)\Upsilon\left(\alpha_2+\frac{1}{2b}\right)\Upsilon\left(\alpha_3\right)\Upsilon\left(\alpha_3+\frac{Q}{2}\right)}{\Upsilon\left(\frac{\delta_1}{2}\right)\Upsilon\left(\frac{\delta_1+Q}{2}\right)\Upsilon\left(\frac{\delta_2}{2}\right)\Upsilon\left(\frac{\delta_2+Q}{2}\right)\Upsilon\left(\frac{\delta_3+b}{2}\right)\Upsilon\left(\frac{b\delta_3+1}{2b}\right)\Upsilon\left(\frac{\delta-b}{2}\right)\Upsilon\left(\frac{b\delta-1}{2b}\right)}+\right.$$
$$\left.+\frac{\epsilon\Upsilon\left(\alpha_1+\frac{b}{2}\right)\Upsilon\left(\alpha_2+\frac{b}{2}\right)\Upsilon\left(\alpha_1+\frac{1}{2b}\right)\Upsilon\left(\alpha_2+\frac{1}{2b}\right)\Upsilon\left(\alpha_3\right)\Upsilon\left(\alpha_3+\frac{Q}{2}\right)}{\Upsilon\left(\frac{\delta_1+b}{2}\right)\Upsilon\left(\frac{\delta_2+b}{2}\right)\Upsilon\left(\frac{b\delta_1+1}{2b}\right)\Upsilon\left(\frac{b\delta_2+1}{2b}\right)\Upsilon\left(\frac{\delta_3}{2}\right)\Upsilon\left(\frac{\delta_3+Q}{2}\right)\Upsilon\left(\frac{\delta}{2}\right)\Upsilon\left(\frac{\delta-Q}{2}\right)}\right]. \tag{70}$$

Reflection properties (57)-(59) now take the form:

$$\mathbf{C}_{(\alpha_1),(\alpha_2),(\alpha_3)}=G_{NS}(\alpha_1)\mathbf{C}_{(Q-\alpha_1),(\alpha_2),(\alpha_3)}, \tag{71}$$

$$\tilde{\mathbf{C}}_{(\alpha_1),(\alpha_2),(\alpha_3)}=G_{NS}(\alpha_1)\mathbf{C}_{(Q-\alpha_1),(\alpha_2),(\alpha_3)}, \tag{72}$$

$$\mathbf{C}^{\epsilon}_{[\alpha_1],[\alpha_2],(\alpha_3)}=G_R(\alpha_1)\epsilon\mathbf{C}^{\epsilon}_{[Q-\alpha_1],[\alpha_2],(\alpha_3)}=G_{NS}(\alpha_3)\mathbf{C}^{\epsilon}_{[\alpha_1],[\alpha_2],(Q-\alpha_3)}, \tag{73}$$

where

$$G_{NS}(\alpha)=\frac{\nu(\alpha)}{\nu(Q-\alpha)}=\left(\frac{\pi\mu}{2}\gamma\left(\frac{1+b^2}{2}\right)b^{1-b^2}\right)^{\frac{Q-2\alpha}{b}}\frac{b^2\gamma(\alpha b-\frac{Qb}{2})}{\gamma\left(-\frac{\alpha}{b}+\frac{Q}{2b}\right)}, \tag{74}$$

$$G_R(\alpha)=\frac{\rho(\alpha)}{\rho(Q-\alpha)}=\left(\frac{\pi\mu}{2}\gamma\left(\frac{1+b^2}{2}\right)b^{1-b^2}\right)^{\frac{Q-2\alpha}{b}}\frac{\gamma\left(\frac{1}{2}-\frac{Qb}{2}+\alpha b\right)}{\gamma\left(\frac{1}{2}+\frac{Q}{2b}-\frac{\alpha}{b}\right)}. \tag{75}$$

When $\alpha=Q/2+ip$; $p\in\mathcal{R}$ (only the states, corresponding to such charges and their descendants contribute to one loop partition function [7]), the functions (73), (74) are called Super-Liouville "reflection amplitudes"

$$S_{NS}(P)=G_{NS}\left(\frac{Q}{2}+iP\right)=-\left(\frac{\pi\mu}{2}\gamma\left(\frac{1+b^2}{2}\right)b^{1-b^2}\right)^{-\frac{2iP}{b}}\frac{\Gamma(1+iPb)\Gamma\left(1+\frac{iP}{b}\right)}{\Gamma(1-iPb)\Gamma\left(1-\frac{iP}{b}\right)}, \tag{76}$$

$$S_R(P) = G_R\left(\frac{Q}{2}+iP\right) = \left(\frac{\pi\mu}{2}\gamma\left(\frac{1+b^2}{2}\right)b^{1-b^2}\right)^{-\frac{2iP}{b}} \frac{\Gamma\left(\frac{1}{2}+iPb\right)\Gamma\left(\frac{1}{2}+\frac{iP}{b}\right)}{\Gamma\left(\frac{1}{2}-iPb\right)\Gamma\left(\frac{1}{2}-\frac{iP}{b}\right)}. \tag{77}$$

Reflection amplitudes $S_{NS}(p)$ and $S_R(p)$ have unit modules, which means, that as in bosonic case we have complete reflection.

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Note Added

After this work was completed I have learned that in ref.[22] three point functions of $N = 1$ SLFT were proposed, extending the method of the paper [16] to the supersymmetric case. It is worth noting, that the generalized special functions $\Upsilon_1(x)$, $\Upsilon_2(x)$, introduced by the authors of [22] and used in the expressions of the three-point functions, in fact, can be expressed via the function $\Upsilon(x)$ as follows: $\Upsilon_1(2x) = \Upsilon(x)\Upsilon(x+Q/2)$, $\Upsilon_2(2x) = \Upsilon(x+b/2)\Upsilon(x+1/2b)$. These relations can be checked directly using the definitions. Performing corresponding substitutions, it is easy to see, that the results, of [22] coincide with the results, presented here in the NS-sector, while in the case of R-sector there are some differences.

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